Prudential Math Challenge 5

Restated Problem
Fill in the missing values of the cash inflows and outflows for the next 10 years and determine the financial feasibility of the Starlink investment by using the equation for NPV.

Assumptions

1. The cash flow for year 2020 corresponds to time period 1, 2021 corresponds to time period 2, and so forth.
2. Satellite launch cost is linearly correlated with number of launches (This will be used to calculate the best fit line later on). Because we did not have the actual cash outflow, we used the limited data that we had.

    *Using linear regression to calculate cash outflow based on the number of launches has limitations as it does not take into account factors like failed landings, changes in landing sites, and improvements in technology over time.
3. Compounding rates behave the same as compound interest. By using $(1+\frac{i}{n})^{nt}$ for the denominator instead of $(1+\frac{i}{n})^{t}$ in the NPV equation, we can account for the rate of return compounding twice per year. Our assumption is justified because without the nt, we would only be compounding once a year.

Analysis
Our team was able to find the cash inflow for both the satellite internet and satellite launch on the Morgan Stanley report. We got the following values:

Cash inflow for satellite internet ($ in millions)	Cash inflow for satellite launch ($ in millions)
0	1,503
1,296	1,999
1,759	2,441
2,258	2,834
2,794	3,182
3,530	3,488
4,275	3,756
5,028	3,988
5,793	4,187
6,570	4,356

Source: Morgan Stanley Research

We could not find the cash outflow for satellite launch. To fix this, we calculated the cash outflow for satellite launch using linear regression based on the given cash outflow values in a year and the number of launches in that year (see assumption 2).

Number of launches	Outflow for launch ($ in millions)
30	2,000
42	1,700
78	3,543
114	2,540
138	4,549

Using linear regression in Excel:

Regression Statistics	
Multiple R	0.8001
R Square	0.6403
Adjusted R Square	0.5204
Standard Error	812.6
Observations	5

	Coefficients
Intercept	1225.7
N launch	20.406

If the outflow for launch ($ in millions) is y and the number of launches is x, then the line of best fit is:

$$y = 20.406x + 1225.7$$

According to the R squared value, the best fit line explains approximately 64% of the data points.

Using the best fit line for the missing cash outflows, the complete table is shown below:

Number of launches	Cash outflow for satellite launch ($ in millions)
30	2,000
42	1,700
54	2,328
66	2,573
78	3,543
90	3,062
102	3,307
114	2,540
126	3,797
138	4,549

*blue = given cash outflow, orange = expected cash outflow

We then calculated the total cash flow for each period:

Total cash flow (CF) = inflow (internet) + inflow (launch) – outflow (launch)

Period	CF_t ($ in millions)
1	-497
2	1,595
3	1,872
4	2,519
5	2,433
6	3,956
7	4,724
8	6,476

9	6,183
10	6,377

Calculating NPV

Unlike the equation given on the competition website, we used nt instead of t to account for the compound interest being applied twice a year (see assumption 3). The equation for NPV should be:

$$\text{NPV} = \sum_{t=1}^{10} \frac{CF_t}{(1+\frac{i}{n})^{nt}} - inital\ investment$$

Plugging in $n = 2$, $i = 0.09$, CF_t, t, and the initial investment:

$$\text{NPV} = \sum_{t=1}^{10} \frac{CF_t}{(1.045)^{2t}} - inital\ investment$$

$$= \frac{-497}{(1.045)^{2(1)}} + \frac{1595}{(1.045)^{2(2)}} + \cdots + \frac{6377}{(1.045)^{2(10)}} - 10000$$

$$= 9188\ (\$\ in\ millions)$$

Starlink should proceed with the investment because the net present value of the investment is $9,188,000,000.

Citations

Morgan Stanley. (2019). *SpaceX, Starlink and Tesla: Moving into Orbit?.* Retrieved March 7, 2020 from https://assets.documentcloud.org/documents/6416324/SPACE-20190917-SpaceX-valuation-Morgan-Stanley.pdf.

Compound Interest Formula. (n.d.). Retrieved March 7, 2020, from https://qrc.depaul.edu/StudyGuide2009/Notes/Savings Accounts/Compound Interest.htm.

Bonus Question

According to Investopedia (https://www.investopedia.com/) , a feasibility study would include economic, technical, legal, and scheduling considerations, what is your analysis to implement a digital literacy program that can impact 100% of El Paso County middle school youth?

Feasibility Study

Technical: By launching new Starlink satellites into space, SpaceX plans to develop broadband internet to a new level, in both efficiency and availability. SpaceX's abundant resources, including the first krypton powered system and integrated solar panels, will only help in developing the borderland. This innovation will utilize technology in a novel manner to spread internet access throughout the planet, especially to areas that require it the most. With the usage of technology in schools dramatically increasing in recent decades, efforts to bridge the gap have been made over time. The wide disparity in income levels and internet accessibility in the El Paso region will be reduced by the implementation of a digital literacy program through Starlink. This product, by mending the differences between the community, is the first of its kind and thus does not face strong competition. However, mandating the digital learning courses accommodated by this new technology in every school will be a challenge to schools.

Scheduling: As mentioned in the official Starlink website, the initial goal of the company is to provide coverage for North America by the end of 2020, and then expand to the rest of the world in 2021. Some of our marketing strategies will include emphasizing its innovative nature in schools and its ability to educate the local children. The specific audience for this program are school districts aiming to improve the access to digital courses. Thus, coordinating a program for every school with Starlink devices is a potential obstacle. However, the technology itself will provide more middle schoolers with access to educational resources and help teach them how to use them through online courses.

Economics: For Tesla to have a net profit from Starlink, it would have to charge school districts for the use of their high-speed internet. Not only is this beneficial to Starlink but to school districts as well since the cost would be relatively cheaper than options from other companies. Therefore, the company's offer would optimally lower income tax (from El Paso citizens) funded for public education. Because several school districts in El Paso already offer computers to their students, Starlink could partner with industries such as Apple, HP, and/or Dell to desing and implement the courses on digital literacy. These digital literacy programs would have a positive effect on El Paso as students would more easily transition to highly-skilled professions, jobs that pay higher salaries. This would in turn increase the development of El Paso as a major urban city, creating more jobs in technological fields. Although these programs would (ultimately) impact students positively, it would require that teachers be tech savvy, which would open up more teaching positions (with different salaries and responsibilities).

Legal: As the internet is relatively new, the government is constantly revising laws so that they also incorporate digital lifestyles. Also, the private sector, including Starlink, cannot decide for the schools themselves as to what to include in the class curriculum. This would pose a potential risk to Starlink, for the company will only have a vague idea of its potential profits from school districts.

Due to the profitability of this program, as well as the community benefit, SpaceX should continue with Starlink to increase the ease of providing digital literacy opportunities to the El Paso region. The market conditions have generally remained the same, and as a result of the novelty of this product, competition is limited.

Citations

https://www.dallasfed.org/-/media/Documents/cd/pubs/digitaldivide.pdf?la=en
https://www.starlink.com/

Round 1 Problem Set

The importance of technology and its benefits as it becomes a larger part in our everyday lives can hardly be exaggerated. Technology has developed and shaped to benefit scientific experiments (through computer simulations and recording devices), the economy and the industries it is comprised of, educational processes, and our recreational activities.

Technology, however, does come at a price; a price that many cannot afford. A study by the Pew Research Center analyzed the median incomes in various cities and provided a demographic breakdown of internet access within certain race and economic groups. The study shows a disparity between the technological access available to each of the following races: White, Black, Hispanic, and Asian.

The problem set asked for an analysis of the percentage of the population that was Hispanic (or Asian, in the second problem), had an internet connection, and had an income above $50,000.

Analysis and Solution

Question 1: In a classroom of 28, what is the probability of selecting a student who is Hispanic that has an internet connection and a household income above $50,000? Assuming 68% of the classroom has an annual household income below $50,000.

Conditions Needed for Student:

A) *They must be Hispanic*

B) *They must have an internet connection*

C) *They must have a household income above $50,000*

To find the percentages for each of those conditions, we must look at the given data and apply the Complement Rule.

A) *The data states that 83.6% of students enrolled in a Texas School District are Hispanic*

B) *Looking at the table, 87.2% of students that are Hispanic and have an income above $50,000 have an internet connection*

C) *Using the Complement Rule, we find that 32% of students have an annual household income above $50,000*

The Complement Rule

$P(A^C) = 1 - P(A)$

P(Household Income ≥ $50,000) = 1 – P(Household Income < $50,000)

= 1 – 0.68

= 0.32

= 32%

Knowing the percentages of the conditions allowed us to apply the General Multiplication Rule to find the probability of all three conditions occurring.

The General Multiplication Rule

P(A and B)=P(A)•P(B I A)

P(*$50,000 income and Hispanic origin* and *Internet Connection*)=P(*$50,000 income and Hispanic origin*)•P(*Internet Connection* given *$50,000 income and Hispanic origin)*

In order to solve for the probability of a student having a $50,000 income and being of a Hispanic origin, we must use the Specific Multiplication Rule (we use the specific multiplication rule for two independent events).

The Specific Multiplication Rule

P(A and B)=P(A)•P(B)

P(*$50,000 income* and *Hispanic origin*)=P(*$50,000 income*)•P(*Hispanic Origin*)

=0.32•0.836

=0.26752

Now that we have the probability of a student having a $50,000 income and being of a Hispanic origin (0.26752), we can revisit the General Multiplication Rule step and plug in our values.

The General Multiplication Rule

P(A and B)=P(A)•P(B I A)

P(*$50,000 income and Hispanic origin* and *Internet Connection*)=P(*$50,000 income and Hispanic origin*)•P(*Internet Connection* given *$50,000 income and Hispanic origin)*

=0.26752•0.872

=0.233277

=**23.328%**

The percentage of students that are of a Hispanic origin and have a household income over $50,000 and have an internet connection is 23.328%, so the probability is **0.23328**.

Question 2: In the same classroom, what is the probability of selecting a student who is White or Hispanic that has an internet connection and a household income above $50,000? Assuming 68% of the classroom has an annual household income below $50,000.
We already calculated the probability for a Hispanic student. For the second problem, we must calculate the probability for a White student and add that to the probability for a Hispanic student.

Conditions Needed for Student:

A) *They must be Hispanic or White*

B) *They must have an internet connection*

C) *They must have a household income above $50,000*

To find the percentages for each of those conditions, we must look at the given data and apply the Complement Rule.

A) *The data states that 83.6% of students enrolled in a Texas School District are Hispanic and that 9.5% of students enrolled in a Texas School District are White*

B) *Looking at the table, 87.2% of students that are Hispanic and have an income above $50,000 have an internet connection and 93.3% of students that are White and have an income above $50,000 have an internet connection*

C) *Using the Complement Rule, we find that 32% of students have an annual household income above $50,000*

The Complement Rule

$P(A^C) = 1 - P(A)$

P(Household Income ≥ $50,000) = 1 – P(Household Income < $50,000)

= 1 – 0.68

= 0.32

= 32%

The solution set from here is similar to that of the first problem. The probability of these conditions applying to a White or Hispanic student are represented by the Specific Addition Rule, which applies as the two races are mutually exclusive.

The Specific Addition Rule

P(A or B)=P(A)+P(B)

P(*White origin with conditions met* or *Hispanic origin with conditions met*)=P(*White origin with conditions met*)+P(*Hispanic origin with conditions met*)

Since we know the probability of the conditions being met for a student of a Hispanic origin (from the first problem), we will solve for the conditions of a student of White origin. Knowing the percentages of the conditions allowed us to apply the General Multiplication Rule to find the probability of all three conditions occurring.

The General Multiplication Rule

P(A and B)=P(A)•P(B I A)

P(*$50,000 income and White origin* and *Internet Connection*)=P(*$50,000 income and White origin*)•P(*Internet Connection* given *$50,000 income and White origin)*

In order to solve for the probability of a student having a $50,000 income and being of a White origin, we must use the Specific Multiplication Rule (we use the specific multiplication rule for two independent events).

The Specific Multiplication Rule

P(A and B)=P(A)•P(B)

P(*$50,000 income* and *White origin*)=P(*$50,000 income*)•P(*White Origin*)

=0.32•0.095

=0.0304

Now that we have the probability of a student having a $50,000 income and being of a White origin (0.26752), we can revisit the General Multiplication Rule step and plug in our values.

The General Multiplication Rule

P(A and B)=P(A)•P(B I A)

P(*$50,000 income and White origin* and *Internet Connection*)=P(*$50,000 income and White origin*)•P(*Internet Connection* given *$50,000 income and White origin)*

=0.0304•0.933

=0.0283632

=2.836%

We now go back to the Specific Addition Rule step and add the two probabilities.
The sum of 0.02836 and 0.23328 is **0.26164**.

Assumptions

Our first assumption was that the percentage of the classroom with an annual household income below $50,000 (68%) was not affected or related to the race of the student; that is, that the probability of a household income being above 50K is the same regardless of ethnicity. This assumption allowed us to apply the percentage in our solutions to both problems when solving for the probabilities of both races having an internet connection and an income above $50,000. It also allowed us to substitute P(B) for P(B I A) in the General Multiplication Rule, which states that P(A and B)=P(A)P(B I A). This substitution was necessary as there was no way to calculate P(B I A) with the given data. The assumption is justified as it was stated as a condition in the problem set.
Our second assumption was that the White and Hispanic populations recorded were mutually exclusive, meaning that no person was both White and Hispanic. Our assumption was justified as the table did not identify populations that were both White and Hispanic. This allowed us to substitute 0 for P(A and B) in the General Addition Rule, which states that P(A or B)=P(A) + P(B) - P(A and B). This substitution was necessary as there was no way to calculate P(A and B) with the given data.
Our third assumption was that the data is still applicable today as the observational study was conducted in 2013. This allowed us to use the data in our solution set.
Our fourth assumption was that the classroom is in a school enrolled in a Texas School District, as the race demographics given are only associated with that group.

Solutions Process

Our initial approach to the first problem was to calculate the percentage of Hispanics with income over $50,000 by adding the percentages of Hispanics in different income brackets over $50,000. After discussion, we realized that this assumption was unjustified as adding percentages of groups with different sizes is not the same as the percentage of the total size. Instead of adding percentages from different income brackets, we decided to use the complete data set, using the percentage of Hispanics lacking a high-speed connection with $50,000+ income.

For our calculations of overall probability, we researched the probability rules and discussed its applications to the problem. We used the multiplication rule when multiple events occur together. For example, we multiplied the probability of being Hispanic and the probability of having $50,000+ income to get the probability of being both Hispanic and having an $50,000+ income. The addition rule was used when multiple events occurred separately. For instance, Hispanic and White are separate races, so we added the percentage of White students with internet connection and $50,000+ income and the percentage of Hispanic students with internet connection and $50,000+ income to get the total percentage for a student that is White or Hispanic.

Conclusion

As a part of the El Paso community, we experience the importance of racial diversity in our everyday lives. However, as the data proves, a large gap persists between the different races in terms of income and other lifestyle factors such as internet connection. As our solution, we found that there was a 23.32% chance of a Hispanic student with a family income greater than $50,000 not having access to the internet, whereas the probability for a white student was 2.83%. The significant difference between the two populations may reflect upon the grades of the students. With the increasing importance of technology in schools, the necessity of having internet connection for the purpose of education is only growing. This places the students without access at a disadvantage in comparison with their peers. Furthermore, the data we received reflects the conditions of the entire nation, so the true conditions in El Paso may be understated due to the higher underrepresented populations and a lower median household income than the national average. It is important that these issues are addressed as soon as possible to provide equal opportunities for students from all backgrounds.

Citations

"Conditional Probability." *Conditional Probability*, Yale University, www.stat.yale.edu/Courses/1997-98/101/condprob.htm.

"Basic Probability Rules." Biostatistics, University of Florida, https://bolt.mph.ufl.edu/6050-6052/unit-3/module-6/.

"Conditional Probability and Independence." Biostatistics, University of Florida, https://bolt.mph.ufl.edu/6050-6052/unit-3/module-7/.

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U.S. Census Bureau. (2019, October 29). U.S. Median Household Income Up in 2018 From 2017. Retrieved from https://www.census.gov/library/stories/2019/09/us-median-household-income-up-in-2018-from-2017.html