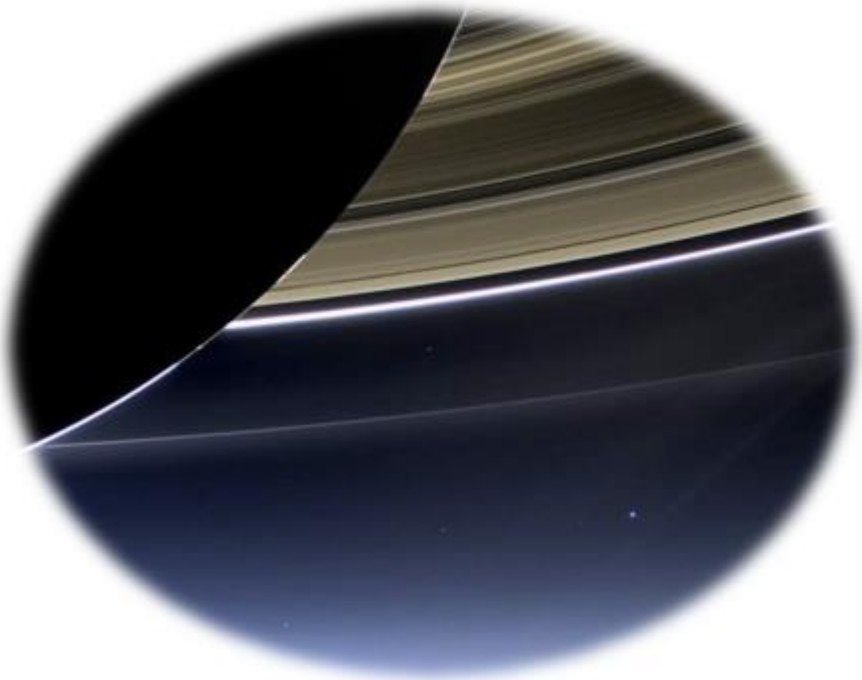
Prudential Math Challenge 5
Round 2

Team Pale Blue Dot
Franklin High School



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I. Executive Summary

SpaceX's newest program, Starlink, is sending a large number of satellites into the lower earth orbit to dramatically increase satellite internet coverage over Northern U.S. and Canada by 2020 and later the entire earth. However, a major factor in question is whether this project is economically viable. To answer this question, the cash in and out flow of SpaceX was determined from the literature, and the net present value (NPV) was calculated. Although the cash-in flow data for satellite internet and launch services were readily available in a Morgan-Stanley financial analysis publication [1], the estimation of missing values of satellite launch business cash-out flow required an extrapolation approach. Based on the limited number of cash-out flow data available, we sought two methods: (i) estimation using a linear trend (an average of 15% and 7% decline in 1-4 and 6-9 years respectively) with every 5^{th}-year cash-out flow increase due to the procurement (or major maintenance) of capital equipment and (ii) estimation using a direct polynomial function curve fitting of available cash-out flow data. The NPV values calculated from this two methods were found to be $28,064,000,000 and $19,950,000,000 respectively. Both values were within the NPV prediction range of Morgan-Stanley. However, we believe the first approach (linear decline with a 5^{th}-year cash-out flow increase) was more logical and accurate. Regardless of the extrapolation method used to estimate cash-out flow, a positive NPV indicates a financially feasible project, and thus SpaceX should proceed with the investment.

II. Background

SpaceX is a company that develops various space systems, and over the years, has become a dominant power in the aerospace industry. Their newest plan will aid in the advancement of the global broadband system. This company's goal is to provide high-speed internet access across the globe and to initiate their plan; they want to target services in the Northern U.S. and Canada this year, finally expanding to global coverages by 2021 [2]. The plan is to launch 42,000 satellites into the lower earth orbit to encompass every region of the world. [3] Therefore, internet providers can buy services from satellite companies, and internet access will become standard in all parts of the world, including the developing countries. If the project is successful, then the divide between digital literacy due to poverty will decrease dramatically, if not, disappear entirely. This project gives an idea of preparing every one of the future generations by providing equal opportunities to everyone despite economic disadvantages.

III. Restatement of the Question

One factor that is raising concerns for Starlink is whether this project is economically feasible and a generally safe investment. To be considered a safe investment or even be profitable, the project will have to surpass the initial amount of $10,000,000,000. The solutions were found through the NPV equation that was provided with the return rate of 9% with only 2 compounding periods per year. To solve the said equation, we must determine the missing values within the cash in-flows satellite internet, launch, and the cash out flow of the satellite launch. If the project is successful, then the next step is to increase the digital literacy of the population to maximize the standardized high-speed internet.

IV. Solution Methodology

To determine whether SpaceX should proceed in investing in the Starlink project, the net present value (NPV), the profitability of the project over the next ten years, needs to be calculated. If the value of the NPV is greater than 0, then the Starlink project will add value to the company if pursued and should, therefore, be accepted. If the value of the NPV is less than 0, then the Starlink project will subtract value from the firm, and SpaceX should therefore not pursue it.

IV.a. Determination of Missing Cash-in Flow for Satellite Internet and Satellite Launch

The missing net cash flow values for satellite internet services can be found in the Morgan-Stanley financial report for SpaceX's Starlink[1]. Similarly, we can fill in the missing values of the satellite launch's cash-In flow by taking the values of revenue from the same document for the years 2020 through 2029. The completed tables with missing values are shown in Figures 1(a) and 1 (b).

Cash-in-Flow For Satellite Internet ($MM)	
Cash-in-Flow 1	0
Cash-in-Flow 2	1296
Cash-in-Flow 3	1759
Cash-in-Flow 4	2258
Cash-in-Flow 5	2794
Cash-in-Flow 6	3530
Cash-in-Flow 7	4275
Cash-in-Flow 8	5028
Cash-in-Flow 9	5793
Cash-in-Flow 10	6570

Cash-in-Flow For Satellite Launch	($MM)
Cash-in-Flow 1	1503
Cash-in-Flow 2	1999
Cash-in-Flow 3	2441
Cash-in-Flow 4	2834
Cash-in-Flow 5	3182
Cash-in-Flow 6	3488
Cash-in-Flow 7	3756
Cash-in-Flow 8	3988
Cash-in-Flow 9	4187
Cash-in-Flow 10	4356

Figure 1 Cash-In Flow (a) Satellite Internet (b) Satellite Launch. Missing Values are in Red

IV.b. Determination/Estimation of Cash-Out Flow for Satellite Launch

The Morgan-Stanley report calculating the profitability of the Starlink project does not provide any detailed information regarding expenditures for the satellite launch services. We were unable to locate additional SpaceX financial documents through an extensive Google web search. Therefore, in order to calculate the outflows of the cash, **the missing values for cash-out flow need to be estimated through an extrapolation of data provided in the Round 2 Question.** We sought two possible approaches for estimating these values: assuming a linear pattern of outflows that increase every five years and a polynomial best fit line.

Cash-out Flow For Satellite Launch	($MM)
Cash-out-Flow 1	2000
Cash-out-Flow 2	1700
Cash-out-Flow 3	1445
Cash-out-Flow 4	1228
Cash-out-Flow 5	3543
Cash-out-Flow 6	3295
Cash-out-Flow 7	2801
Cash-out-Flow 8	2540
Cash-out-Flow 9	2159
Cash-out-Flow 10	4549

Figure 2 Cash-Out Flow for Satellite Launch. Approach I. Estimated Values are in Red

IV.b(i). Approach I: For this first approach, a linear function pattern [**Assumption 1**] was assumed. In Figures 2 and 3, the difference between the cash outflow values for years one and two is a decrease in 15% from $2,000,000,000 to $1,700,000,000. Following this pattern, the cash outflow values for years three and four would be $1,445,000,000 and $1,228,000,000, respectively. However, there was a sudden increase in the cash outflow in year five, increasing to $3,543,000,000. We restart the pattern by finding the slope of cash-

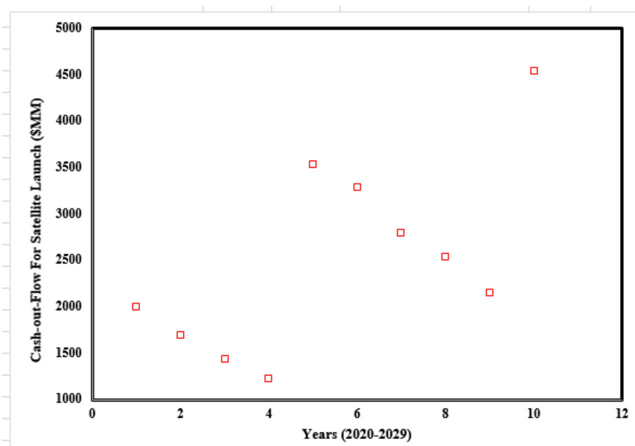

Figure 3 Trend for Cash-Out Flow for Satellite Launch. Approach I

out flow between years five and eight, a 7% rate of decrease [**Assumption 2**]. This trend continued to calculate the cash outflow values for years six, seven, and nine. The linear pattern would increase the cash outflow value of year ten if the pattern was followed. It was assumed that the influx could be because of the increased expenditures related to major maintenance and/or new procurement of capital equipment. Figures 2 and 3 show the cash-out flow trend and completed table, respectively.

Cash-out Flow For Satellite Launch	($MM)
Cash-out-Flow 1	2000
Cash-out-Flow 2	1700
Cash-out-Flow 3	2356
Cash-out-Flow 4	3207
Cash-out-Flow 5	3543
Cash-out-Flow 6	4067
Cash-out-Flow 7	4100
Cash-out-Flow 8	2540
Cash-out-Flow 9	5678
Cash-out-Flow 10	4549

Figure 5 Cash-Out Flow for Satellite Launch. Approach II. Estimated Values are in Red

IV.b (ii). Approach II: For the second approach, a best-fit polynomial [**Assumption 3**] was created based on the limited number of cash-out flow data provided. This line takes the points given and generates trends based on what kind of function one is looking for. This analytical tool can show data movements over periods, in this case, the cash outflow values, and creates a polynomial regression line between variables. We chose a 4th order polynomial curve [Assumption 4] to represent the cash-out flow data. It yielded an R^2 value of 1. The R^2 measures how close the data points are to the regression line on a scale from 0 to 1. Figures 4 and 5 show the cash-out flow trend using this method and the completed table, respectively.

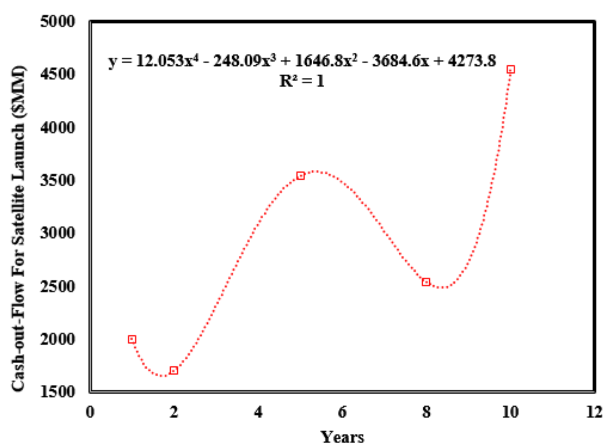

Figure 4 Trend for Cash-Out Flow for Satellite Launch. Approach II

IV.c. Calculation of the Net Present Value (NPV)

After calculating the cash outflow, the net cash flow was calculated by subtracting the cash outflow value from the cash inflow value for each year. The equation for the net present value is shown below.

$$\text{NPV} = \Sigma \frac{CF_t}{(1+\frac{i}{n})^t} - initial\ investment$$

The cash flows are calculated for each year, and they are then divided by one plus the rate of return, *i* and in our case a 9% rate of return, over the amount of compounding periods, *n* and in our case 2 compounding periods, to the power of the year that cash flow belongs to. Finally, the initial investment is subtracted from the summation. The NPV with the first equation.

The NPV values calculated using extrapolation Approach I and Approach II and were found to be **$28,064,000,000** and **$19, 950,000,000** respectively.

IV.d. Analysis

The both values ($28,064,000,000 and $19, 950,000,000) were within the NPV prediction range of Morgan-Stanley (Bear Case: $5,228,000,000, DCF Valuation $52,736,000,000, and Bull Case: $121,176,000,000) [1]. As indicated in Ref [5], the three analyses were based on three distinct scenarios: " 1. Starlink fails completely, and SpaceX remains a launch company. 2. Starlink succeeds but SpaceX captures no more than 10% of traditional broadband users. 3.

Starlink succeeds, and SpaceX breaks into the traditional broadband internet market." The calculated NPV values in this solution paper fall between the bear case (scenario 1) and DCF Valuation (scenario 2).

We believe that the first approach (linear decline with a 5th-year cash-out flow increase) is more logical and accurate. A 5th-year cash-out flow increase is understandable in aerospace industries since higher reliability and safety margin of space systems often required major maintenance or procurement of new capital equipment in every few years. A polynomial trend may also be possible due to an increase in maintenance expenditures as equipment approach their end of service life. However, without additional actual cash-out flow data points of satellite launch, it is difficult to rationalize that a polynomial trend will best represent the data. We looked into some cash-flow information available for aerospace industries in Ref [6]. However, we could not reach a definite conclusion.

Regardless of the extrapolation method used to estimate cash-out flow, a positive NPV indicates a financially feasible project, and thus SpaceX should proceed with the investment.

IV.e. Answer to the Bonus Question

If Starlink's project was economically feasible, this would mean an immense amount of improvement to the current digital divide caused by an economic divide. As the goal of the project is to provide high-speed internet to all the regions on earth, this feat will standardize the reach of high-speed internet, giving equal opportunity to everyone (especially populations in poverty). Not only will the digital divide decrease, but the need for digital literacy in the Borderplex will also increase exponentially. The American Library Association's Digital-Literacy Task Force defines digital literacy as "the ability to use information and communication technologies to find, evaluate, create, and communicate information, requiring both cognitive and technical skills." [2] To ensure our generation is capable of meeting this criteria, many policies must be implemented at the regional, district, and school level. One method to increase digital literacy in the region is ensuring (El Paso) access to 5G. 5G allows data and services to be transferred and downloaded at a faster rate despite a large number of users. This level of the internet will accommodate for the drastic increase in internet users as the economic inequality is addressed. Therefore having 5G in this region can significantly aid in keeping the internet less expensive and running at a higher speed.

Another possible policy that should be implemented is better access to computers throughout the Borderplex region. Every student should be provided with a laptop purposed for their education. As students, especially those who are not financially stable, gain more access to technology, they will need to learn how to create, consume, and communicate media. Another way to improve digital literacy is to implement additional classes that instruct students on how to utilize their electronics and the internet to its fullest extent. These classes would include after school programs such as a Robotics club and Technological Student Association organization. Without proper technology-based education, students will struggle during higher education and throughout daily life as the digital age continues to become more common. For these resources and classes to become feasible, we would likely need an initial investment upwards of 50 million. This entire process would likely span over five years as the program expands. The plan is to have technology-based programs running in every Borderplex district by 2025.

IV.e. Concluding Remarks

Assuming that the cash outflow followed the linear pattern previously mentioned, Starlink's project is projected to be successful by breaking even and producing profits. Using various models to determine and fill in the rest of the table, we created our own data points with a linear pattern, utilizing the slope formula. For the second method, the trendline in excel was used to create an accurate equation, which was used to fill in the chart. With this data, we found the net cash flow through the provided formula to see that the NPV was positive. Due to the methods mentioned previously, the end net cash flow is indeed positive which concludes that the project is profitable be continued.

Citations

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Prudential Math Challenge 5
Round 1

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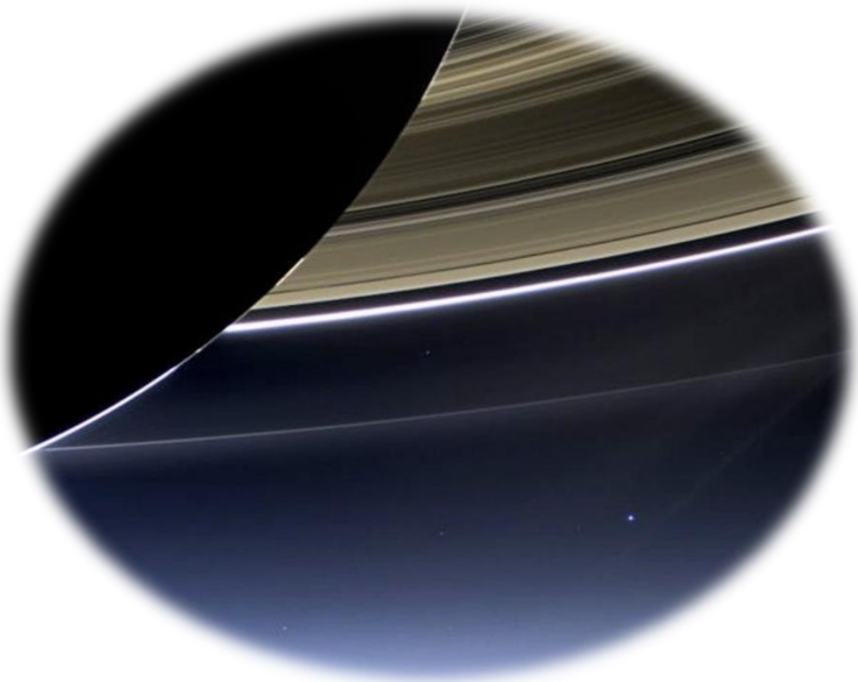

February 15, 2020

Executive Summary

Broadband internet access is important to the world because the internet is a significant part of our lives. It can aid business people, enables the economy to thrive, as well as play a vital role in the educational, workforce, and medical developments. Students who cannot afford the internet due to low-income are at a disadvantage and have to find ways to use the internet. So, our solution, presented later in the essay, helps to highlight this pressing issue. We found that in a classroom of 28 students in the Texas School District, the probability of selecting a Hispanic student with internet connection and an annual household income over $50,000 is approximately 25%, and the probability of selecting a Hispanic student or a White student with internet connection and an annual household income over $50,000 is approximately 29%. To make this conclusion, we had to assume that the distribution of income in the classroom is applied, in the same manner, to the whole district.

Background

Broadband access has transformed society as a whole including the method of education and working environments. However, a large part of the U.S. continues to lack what has become a necessity in today's world. Internet access allows students to submit assignments, spurs innovations, collaborations, and allows for greater economic growth and activity. In addition, the need for the internet has grown exponentially across the national, state, and district levels.

Though, there are many factors that contribute to the lack of accessibility, racial and income variables [Figures 1-2] often come into play [1-3]. By improving these disparities, social and economic mobility can be increased, which is essential to be the betterment of the students in the United States. In order to thoroughly analyze these factors and their impact on broadband access, the following questions were investigated and interpreted.

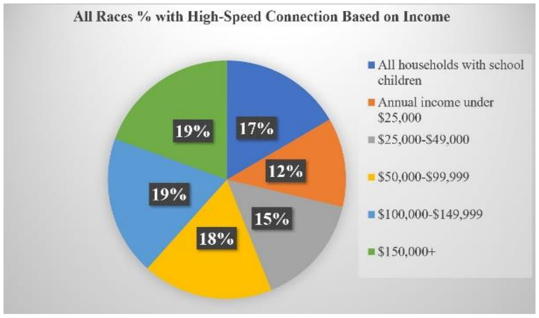

Figure 1. All races (%) with high-speed connection

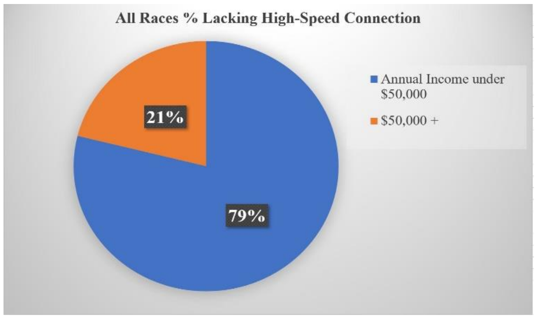

Figure 2. All races (%) lacking high-speed connection

Restatement of the Question

Several observations were made using the data provided by The Pew Research Center [2]. One was the large gap of accessibility seen between income groups. For example, those who fell into the lower income groups (those who made under $50,000) had a severely lower percentage of access to the internet in comparison to those who made over $150,000. Not only did the income level influence their accessibility, ethnicity is another impactful component. Those of Caucasian and Asian nationality tend to have a higher percentage of broadband access in comparison to those of African American and Hispanics [2]. Moving down to the state level (Texas), the percentage for each race changes as the dominant race is the Hispanic nationality at 83.6% while the remaining 16.4% consists of African American, Asians, and Caucasians [4-5].

Keeping these observations in mind, we are able to create our solutions to the problems provided. First, we start with the calculation of the ratio of Hispanic students where their parents earn over $50,000 per year in a classroom of 28 where 68% of the students' income are smaller than $50,000. Lastly, from that same classroom of 28 students, the probability of choosing a White or Hispanic student who has internet percentage with a household income that is above $50,000 is found. This is keeping in mind that 8% of the classroom would have an annual household income that is below $50,000.

Solution, Methodology, and Analysis

Answer to Question 1.0

The probability of a student in the classroom having an annual household income above $50,000 is 32%. **We assume that the distribution of annual household income applies in the same manner to the entire Texas School District.** Of the entire school district's population, 83.6% are Hispanic (H). So, for the classroom sample, the probability of Hispanics having an annual household income above $50,000 is shown as P(A).

$$P(A) = 32\% \ x \ 83.6\% \qquad (1)$$
$$= 26.75\%$$

From the given Table 2, we know that 87.2% of Hispanic households with school-age children and an annual household income of above $50,000, have internet access. So, the probability of choosing a Hispanic student with internet connection and an annual household income above $50,000 from the classroom sample is represented by P(B).

$$P(B) = P(A) \ x \ 87.2\% \qquad (2)$$
$$= 23.32\%$$

Therefore, the probability of selecting a student who is Hispanic and has an internet connection and an annual household income above $50,000 is seen below for P(H).

$$P(H) = \frac{(P(B) \ x \ 28)}{28} \qquad (3)$$
$$= \frac{6.5}{28} \approx \frac{7}{28} \approx \mathbf{25}\%$$

The probability of selecting a student who is Hispanic and has an internet connection and an annual household income above $50,000 is approximately 25%.

Answer to Question 2.0

The probability of a student in the classroom having an annual household income above $50,000 is 32%. **We assume that the distribution of the annual household income applies in**

the same way to the entire Texas School District. Of the entire school district's population, 83.6% are Hispanic and 9.5% are White. So, in prior c, we found that the probability of selecting a student who is Hispanic, has an internet connection, and household income above $50,000 is 25%. For the white students, we will apply the same method.

The probability of White (W) having an annual household income above $50,000- assuming that the classroom sample's income distribution applies in the same way to the district- is 9.5% x 32% = 3.04% (Based on Eqn. 1).

From Table 2, we know that the probability of Whites having an internet connection and an annual household income above $50,000 is 93.3% (Based on Eqn. 2).

Therefore, the number of white students in the classroom sample with internet connection and an annual household income above $50,000 = 32% x 9.5% x 93.3% x 28 = 0.794 ≈ 1. Therefore, the probability of selecting a student who is white, has an internet connection, and a household income above $50,000 is approximately $\frac{1}{28}$ = 3.57% = P (W) (Based on Eqn. 3).

We assume that the two events are mutually exclusive. Therefore,

$$P\ (H \cup W) = P(H) + P(W) \tag{4}$$

$$= \mathbf{3.57\%} + \mathbf{25\%} = \mathbf{28.75\%} \approx \mathbf{29\%}$$

The probability of selecting a student who is White or Hispanic that has an internet connection and a household income above $50,000 is approximately 29%.

Concluding Remarks

If we assume that the district's annual household income distribution is the same as the classroom's income distribution, we can conclude that the probability of selecting a Hispanic student with an internet connection and an annual household income of $50,000 is approximately 25%. We can also conclude that the probability of selecting either a Hispanic student or White student with an internet connection and an annual household income of $50,000, assuming they are mutually exclusive events, is approximately 29%.

Citations

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